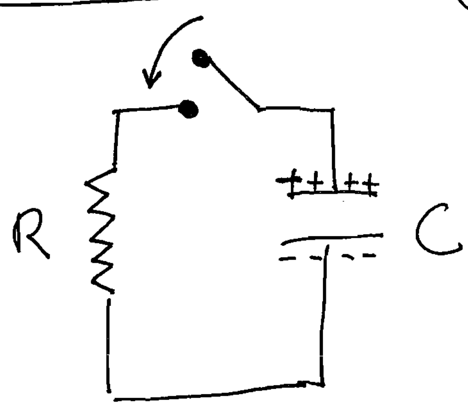
RC Time-Domain Analysis:

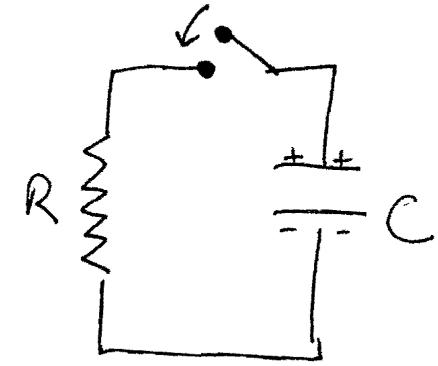
In general time domain analysis employs Kirchoff's energy conservation rule to create a differential equation which we need to solve.

RC Discharge Time:



Consider a capacitor that is charged with Q. When you close the switch and complete the circuit how does the charge on the capacitor behave as a function of time?

Solution :



After closing the switch, At some instant in time the voltage across the resistor equals the voltage across the capacitor. (Measurement polarity is important)

1 $\underbrace{V_R = V_c}$ where $V_R = IR$, $V_c = \frac{Q}{C}$

So 1 becomes $\frac{Q}{C} = IR$

2 Now $I = \frac{-dQ}{dt}$ (loosing charge)

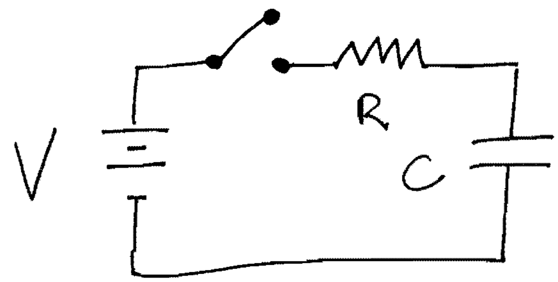
$\Rightarrow \frac{Q}{C} = -R\frac{dQ}{dt}$ or $\boxed{\frac{dQ}{dt} = -\frac{1}{RC} Q}$

$\Rightarrow \boxed{Q(t) = e^{-t/RC}}$

Recall $I(t) = \frac{dQ(t)}{dt}$

and $V_c(t) = \frac{Q(t)}{C}$

RC Charging Time :



Now charge the "empty" capacitor with a battery. what is $Q(t)$?

(1) $\sum_{\substack{\text{closed}\\ \text{loop}}} \Phi = 0 = V - IR - \frac{Q}{C}$

(2) Substitute $I = \frac{dQ}{dt}$; $V - R\frac{dQ}{dt} - \frac{Q}{C} = 0$

Rearrange . . .

$$\frac{dQ}{dt} = \frac{CV - Q}{RC} \Rightarrow \int_0^Q \frac{dQ'}{Q' - CV} = \int_0^t \frac{-dt}{RC}$$

$$\ln(Q' - CV)\Big|_{Q'=0}^{Q'=Q} = \frac{-t}{RC}$$

$$\frac{Q - CV}{-CV} = e^{-t/RC} \quad \text{or} \quad \boxed{Q(t) = CV\left(1 - e^{-t/RC}\right)}$$